

Paths, trees and asteroids

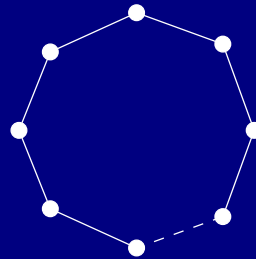
Benjamin Lévêque

Chordal graphs

Chordal graphs

Definition

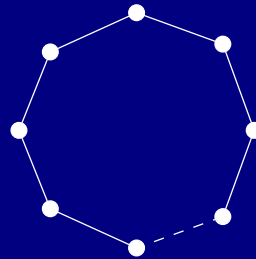
Graphs with no holes



Chordal graphs

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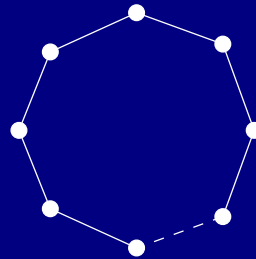
Theorem (Gavril - 1974)

Chordal \iff intersection graph of a family of subtrees of a tree

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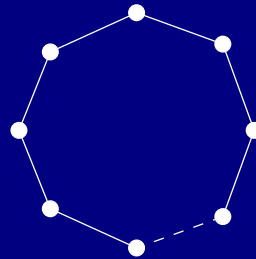
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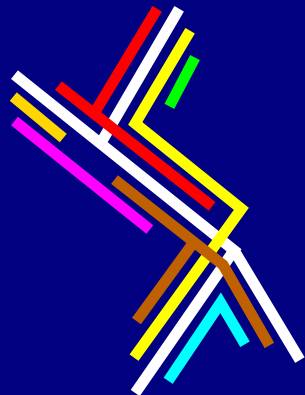
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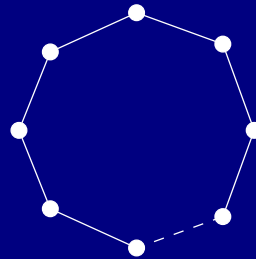
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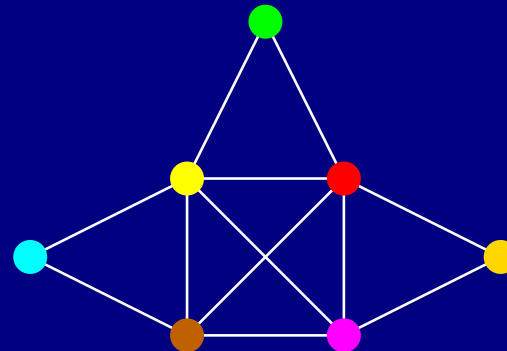
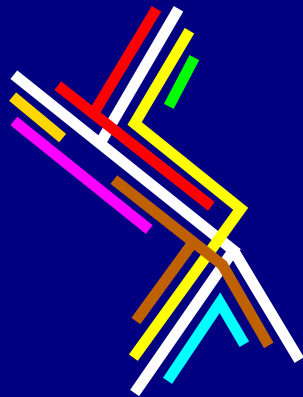
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Interval graphs

Interval graphs

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Intersection graph of a family of subpaths of a path

Interval graphs

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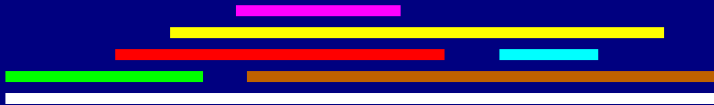
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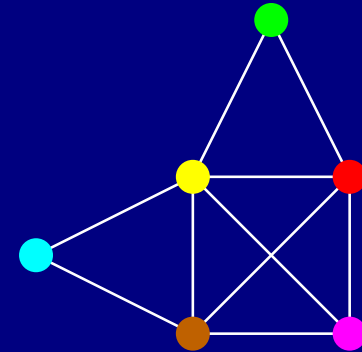
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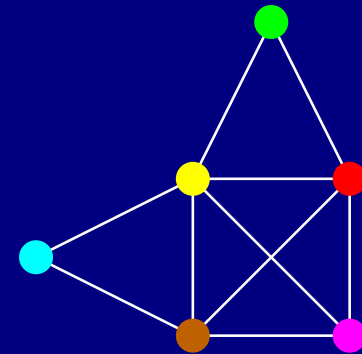
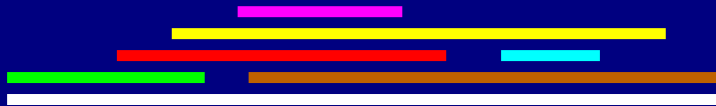
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Interval graphs

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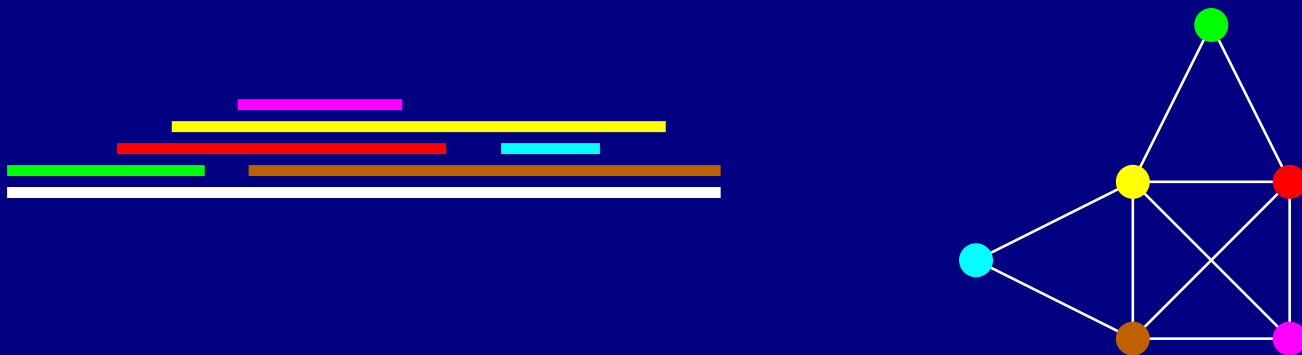
Asteroidal triple

3 non adjacent vertices such that between any two of them, there exists a path that does not intersect the neighborhood of the third.

Interval graphs

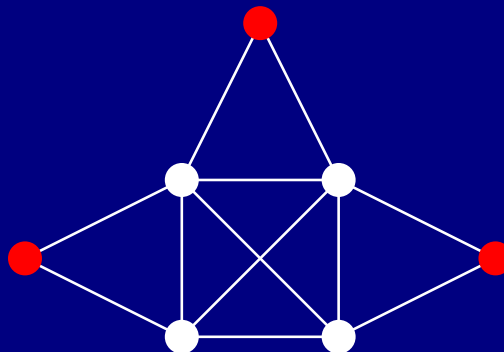
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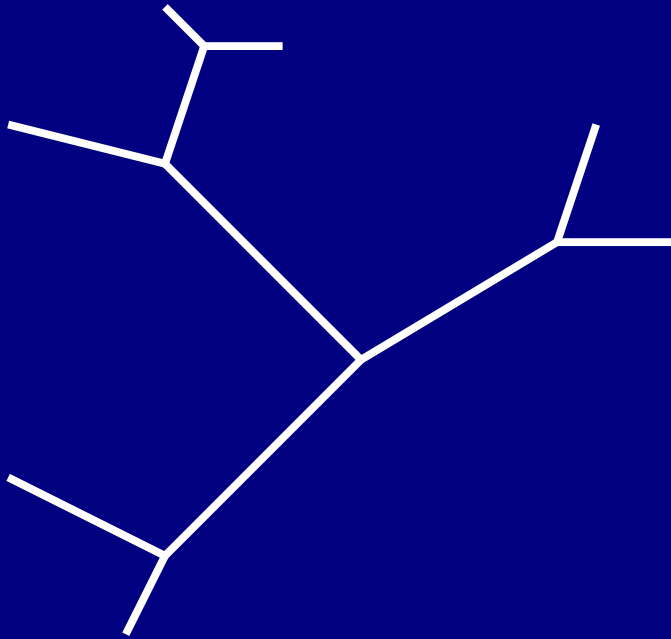
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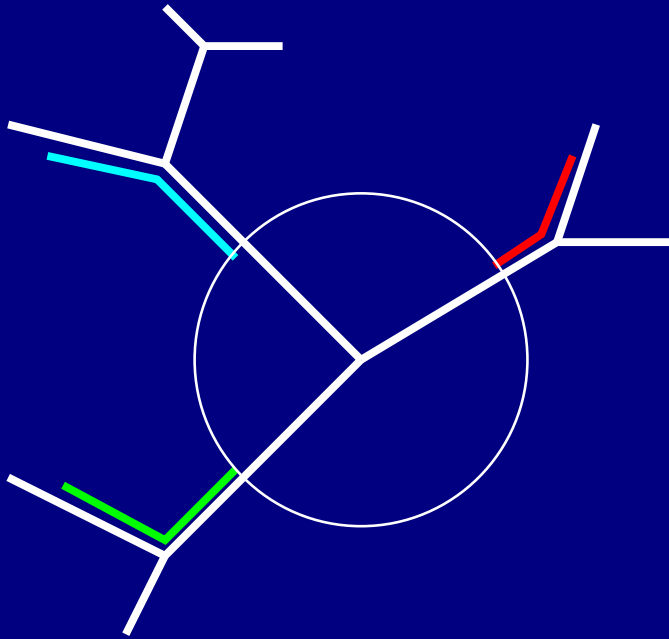
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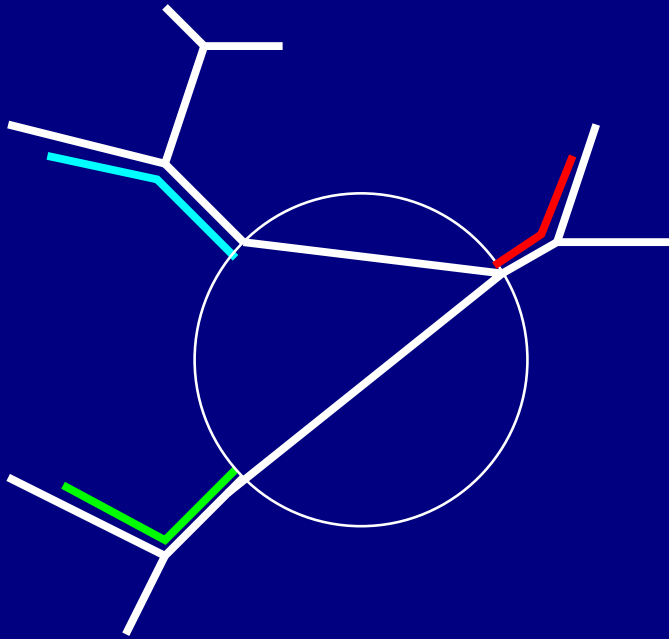
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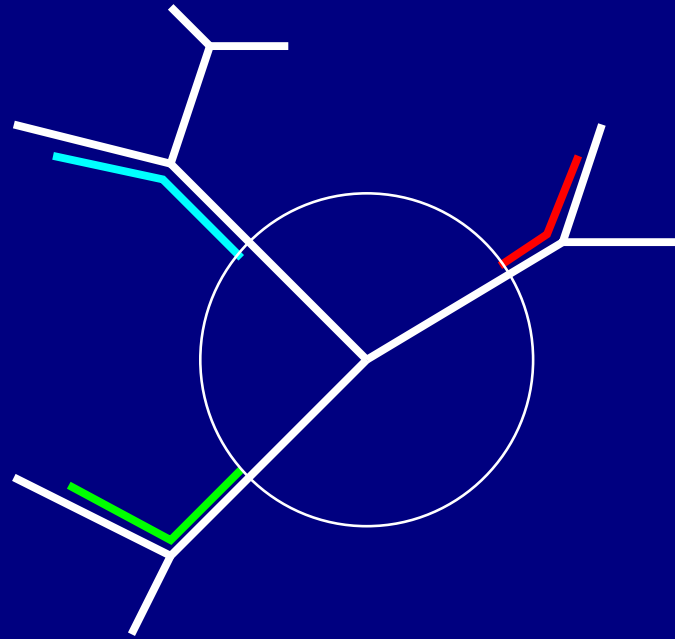


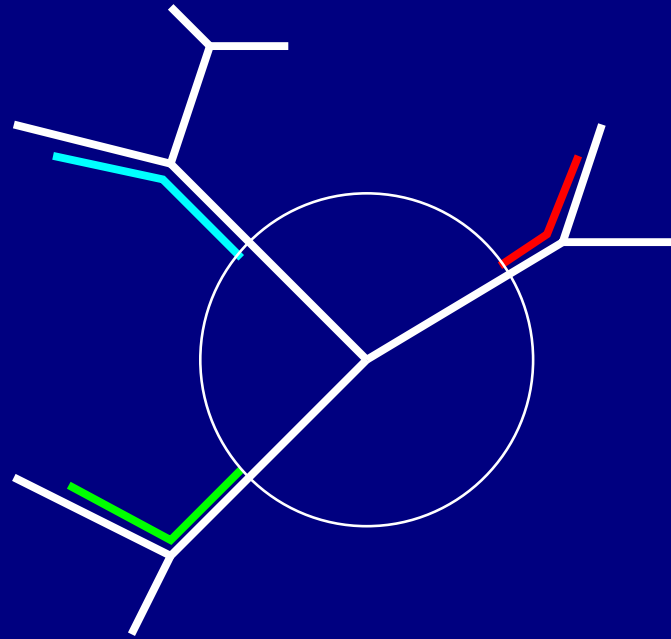






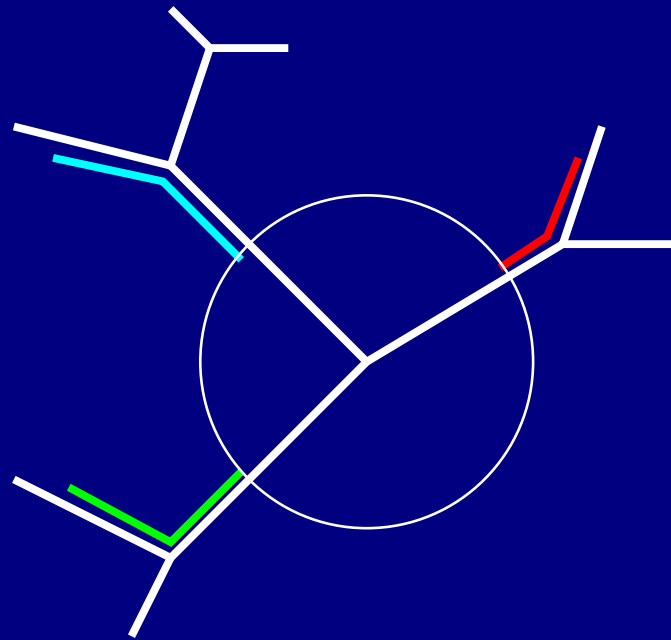






Theorem (Lekkerkerker and Boland - 1962)

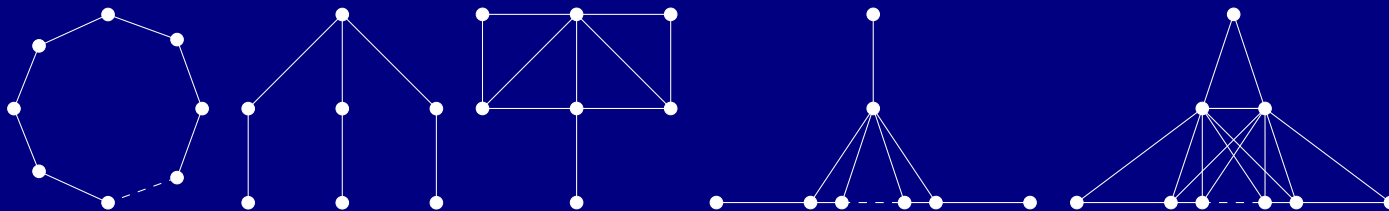
Interval graph \iff chordal + no asteroidal triple



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Path graphs

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Intersection graph of a family of subpaths of a tree

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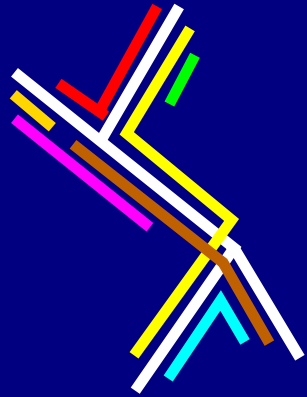
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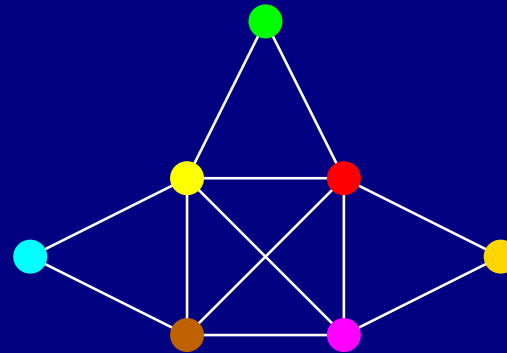
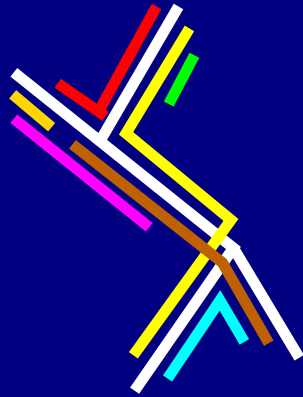
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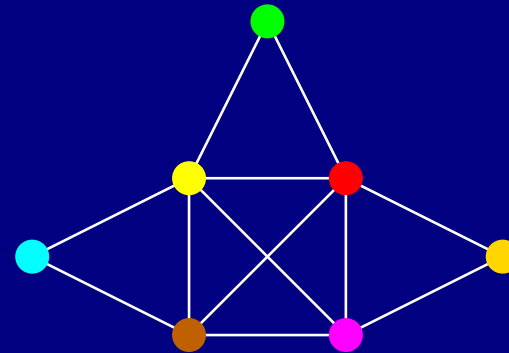
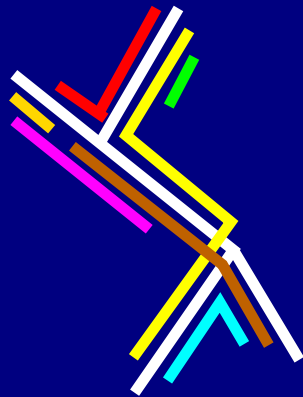
Intersection graph of a family of subpaths of a tree



Path graphs

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Intersection graph of a family of subpaths of a tree



Question (Renz - 1970) :
Characterization by forbidden induced subgraphs ?

Neighborhoods in path graphs

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Remark

The neighborhood of every vertex is an interval graph

Neighborhoods in path graphs

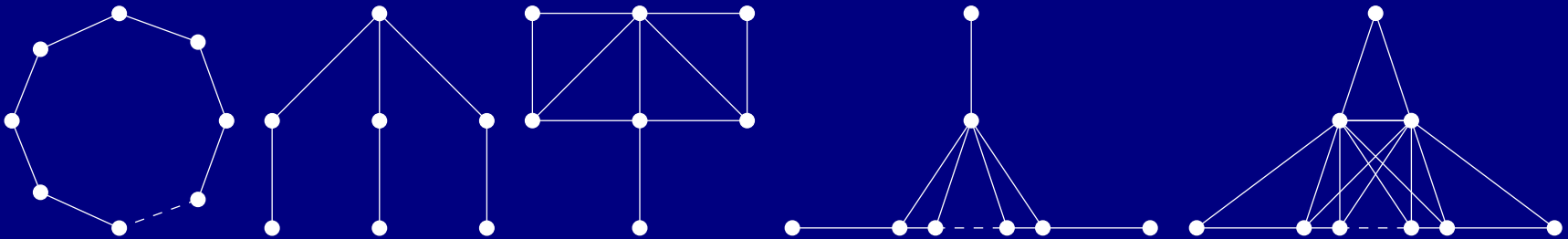
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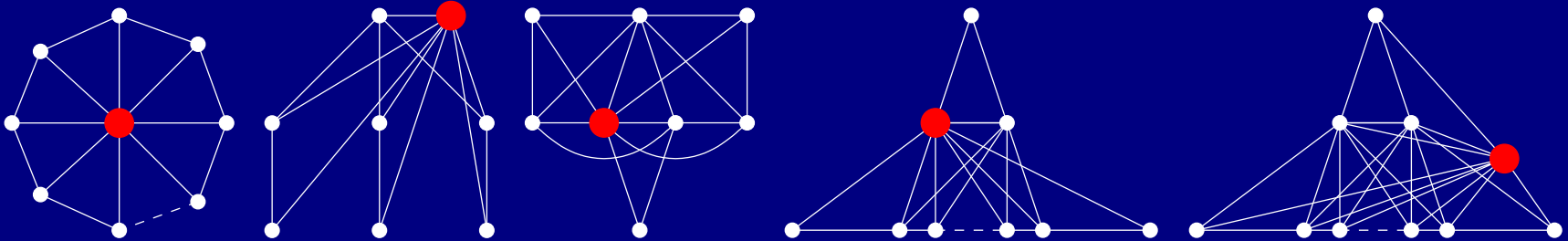
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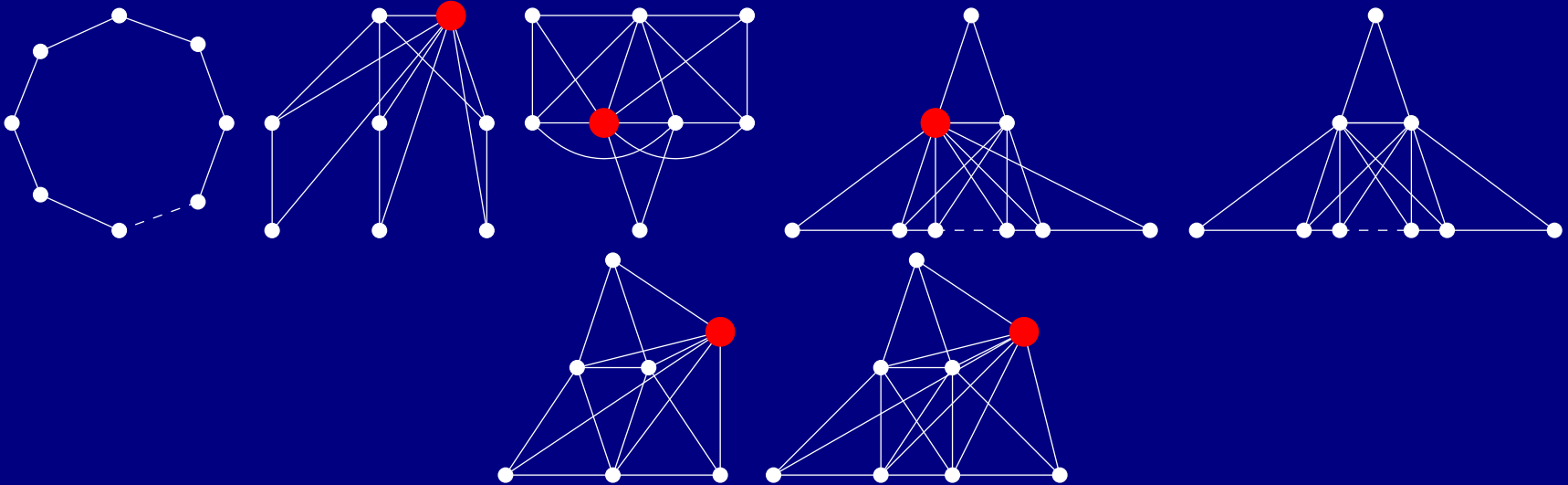
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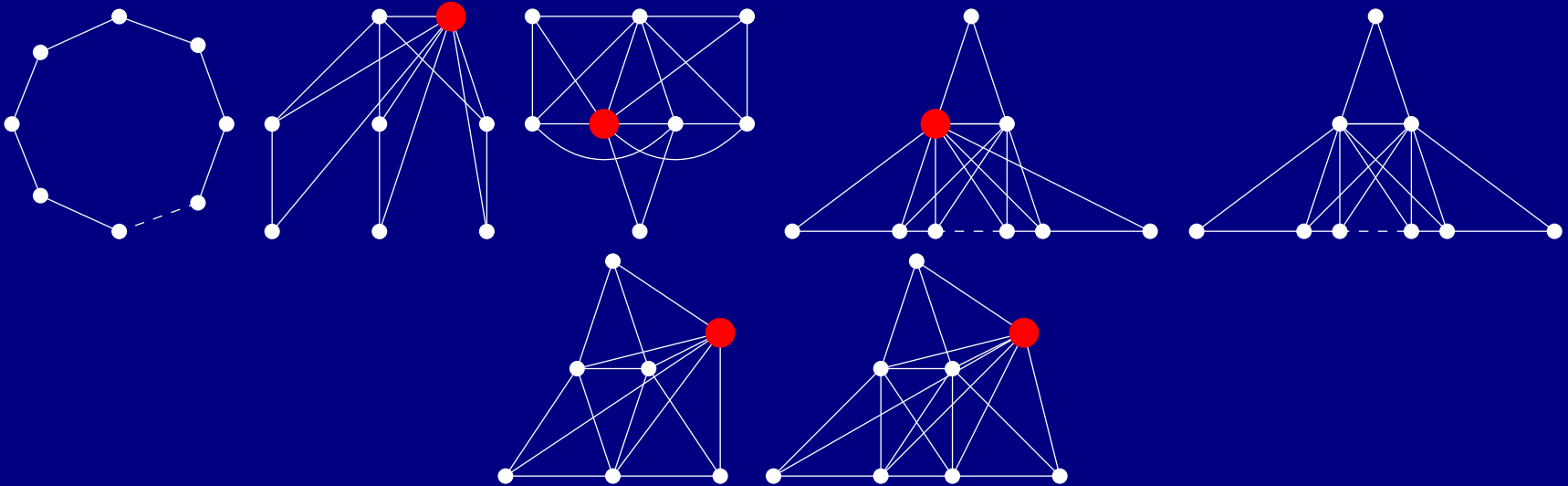
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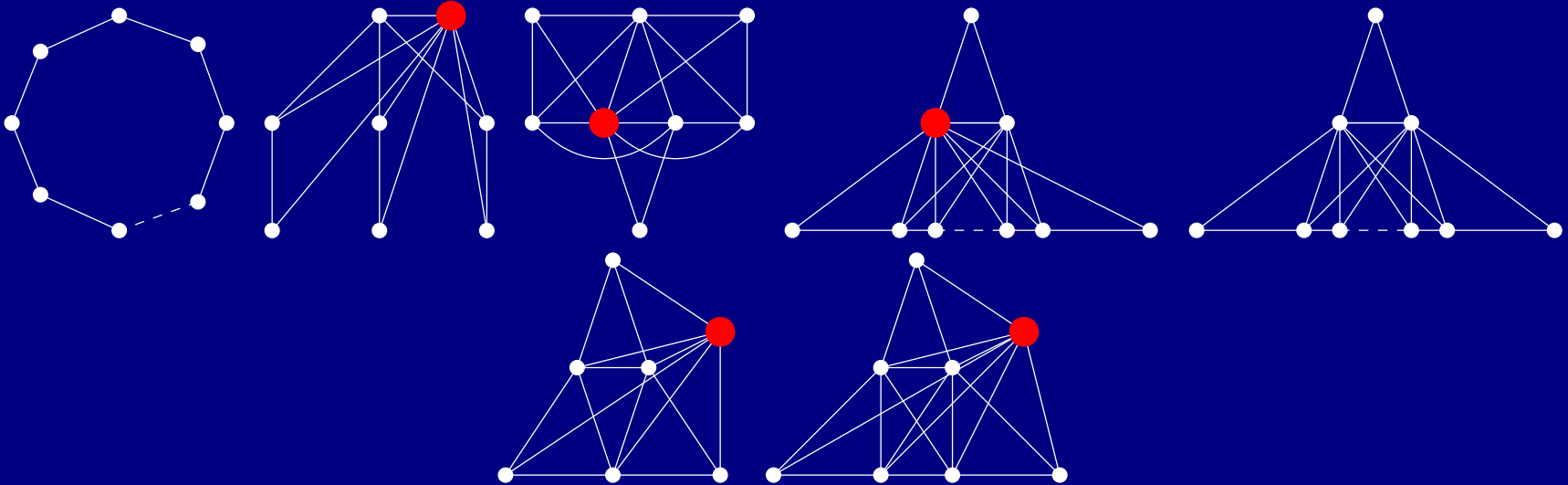


Question : Is that enough ?

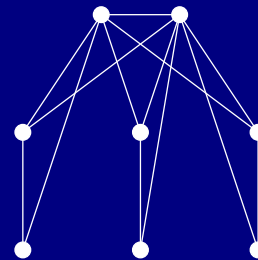
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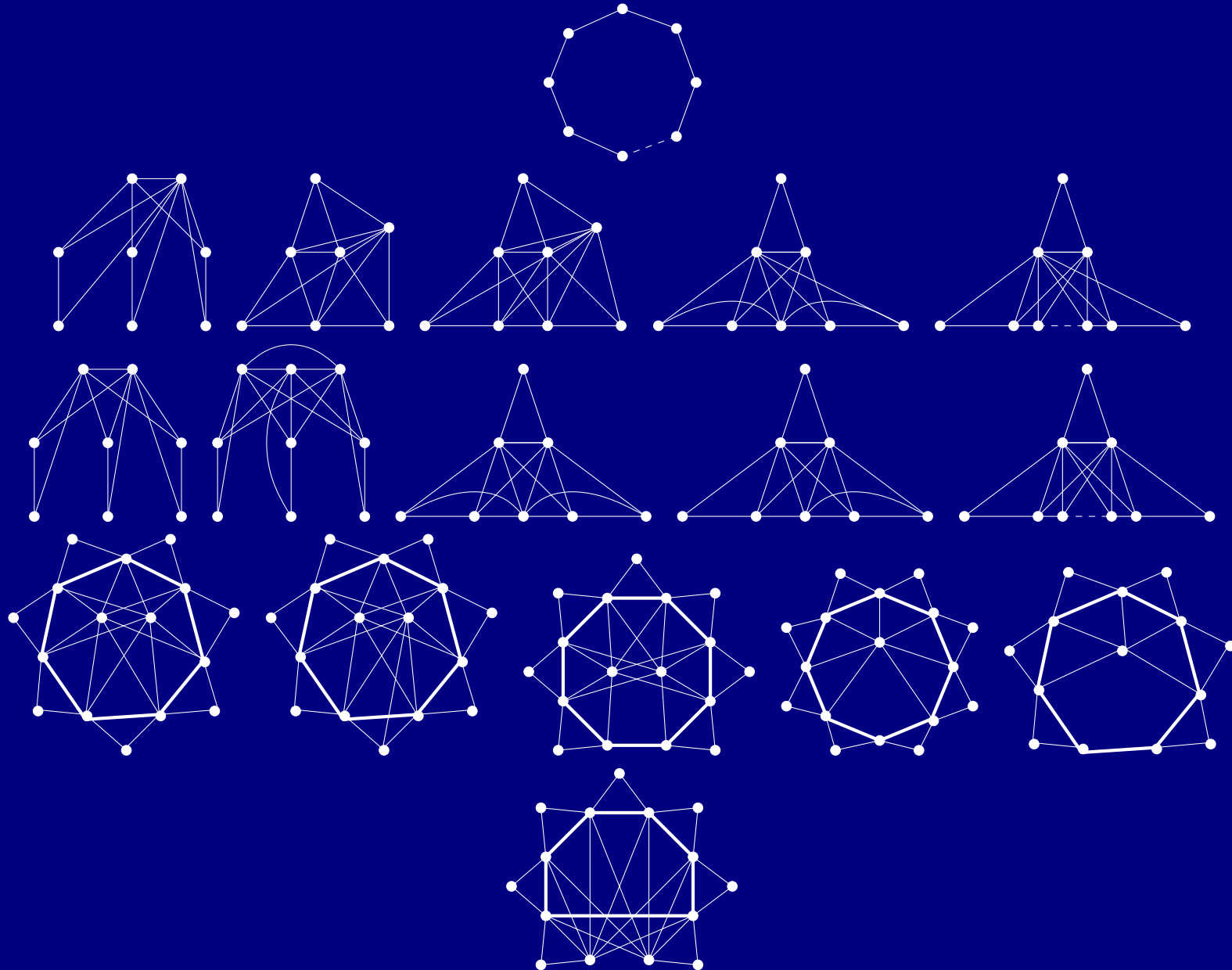
(Renz - 1970)

Theorem (Lévêque, Maffray and Preissmann - 2009)

Path graph \iff it does not contain :

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Simplicial vertices

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Chordal, not a clique \implies 2 non adjacent simplicial vertices

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Generalization (Lévêque, Maffray and Preissmann - 2009)

Chordal, not a clique \implies 2 non adjacent "special" simplicial vertices
($N[v] \cap N(V \setminus N[v])$ is a maximal separator)

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Simplicial vertices

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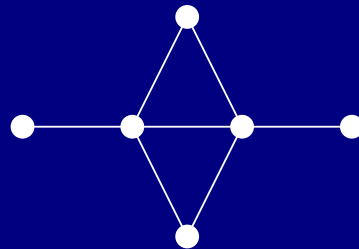
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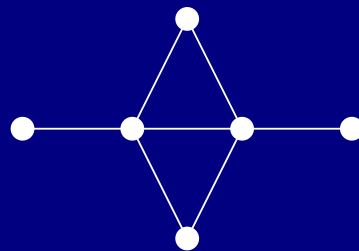
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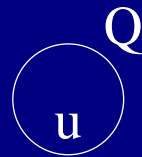
Question : "special" simplicial elimination ordering in linear time ?

Proof

Suppose there exists a graph minimally not path graph not in the list

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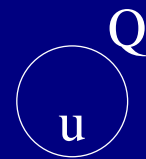
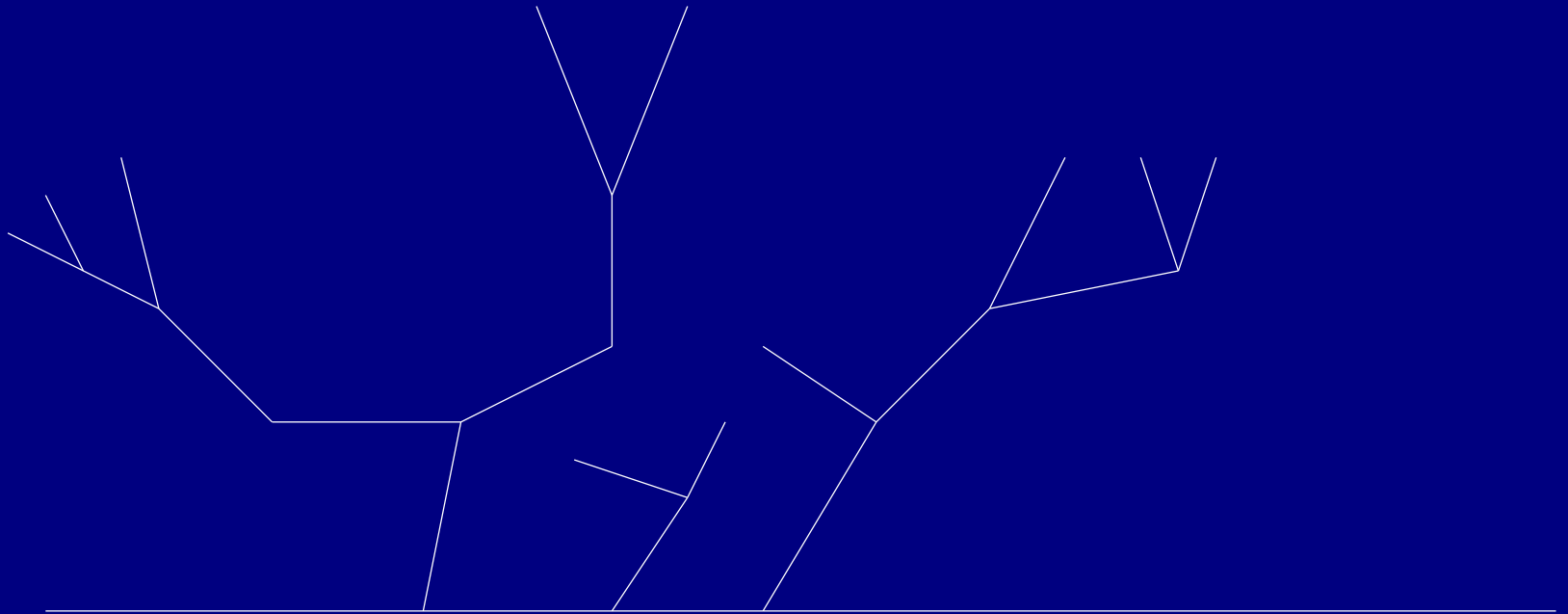
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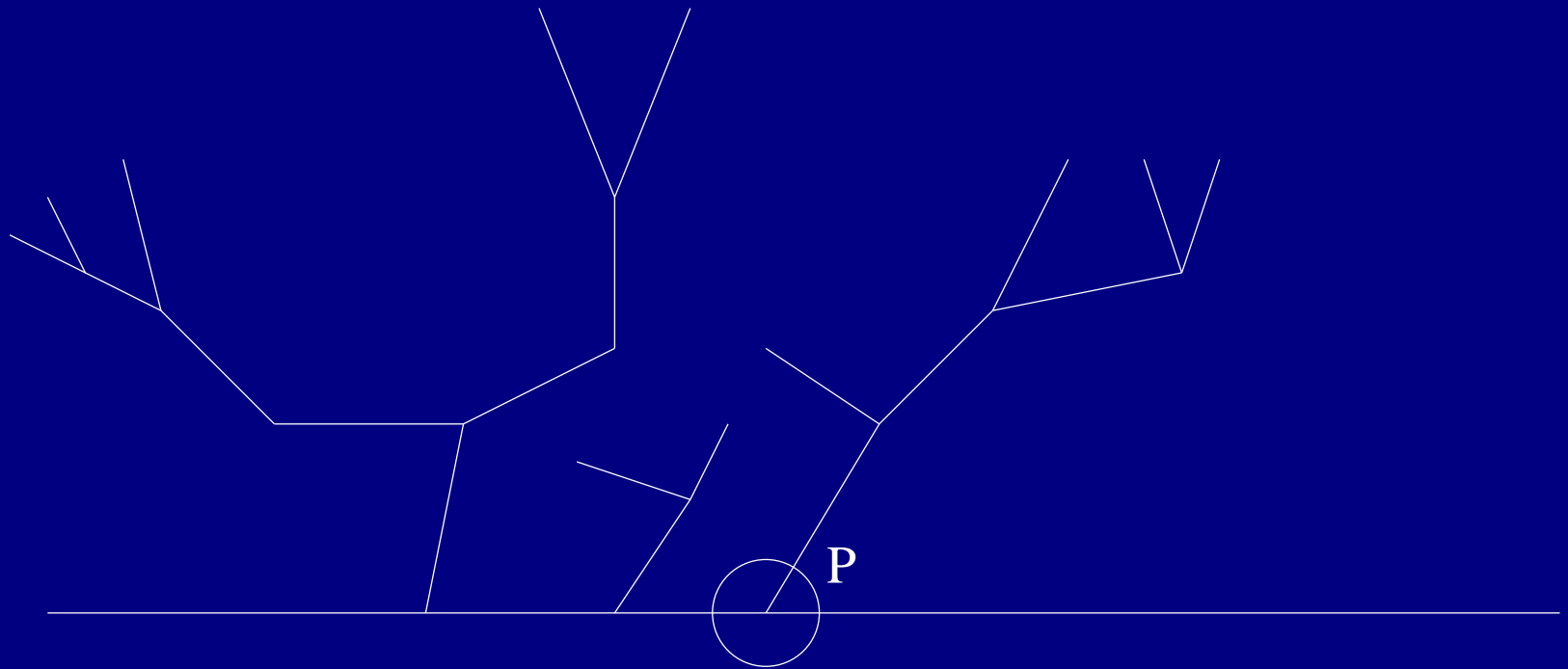
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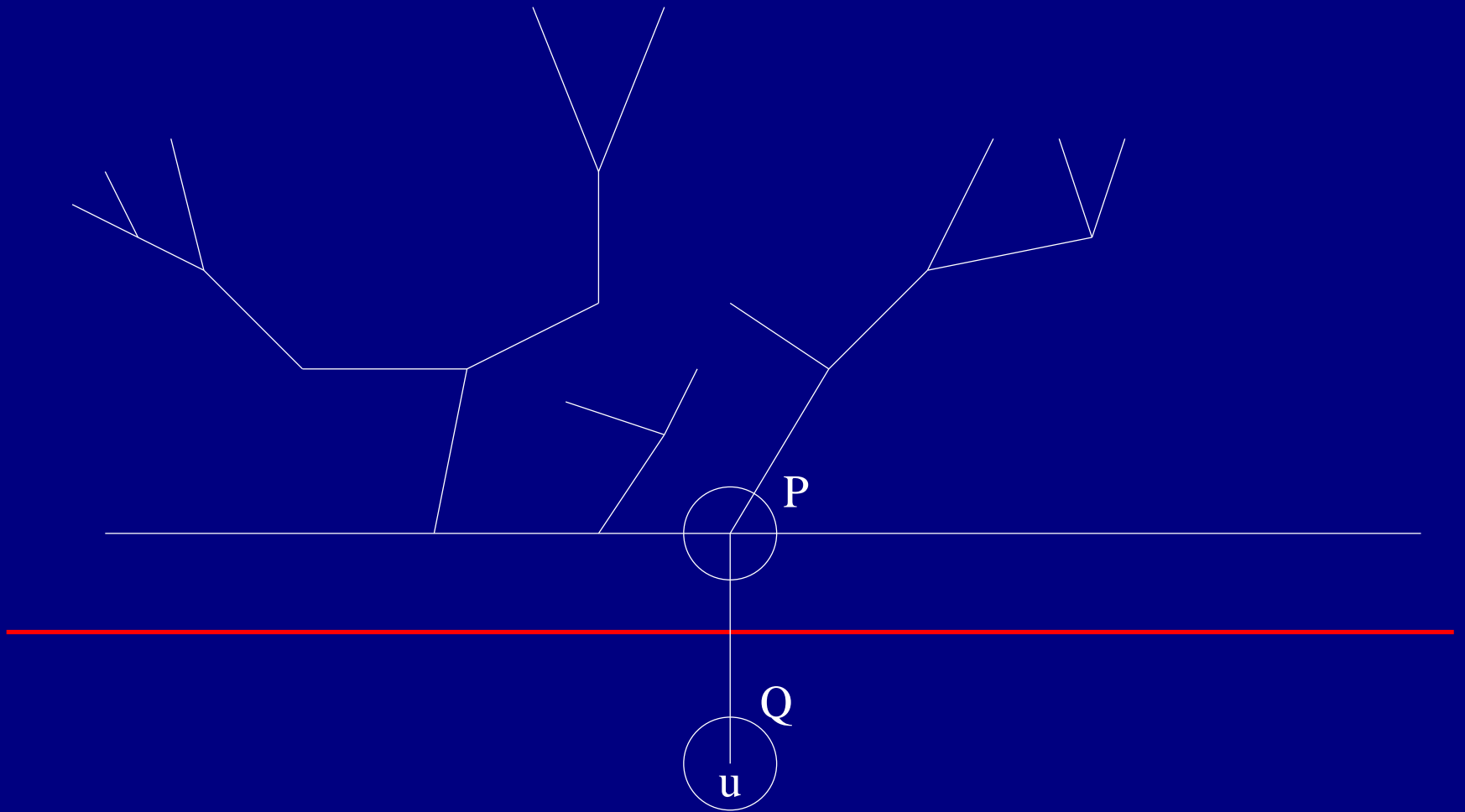
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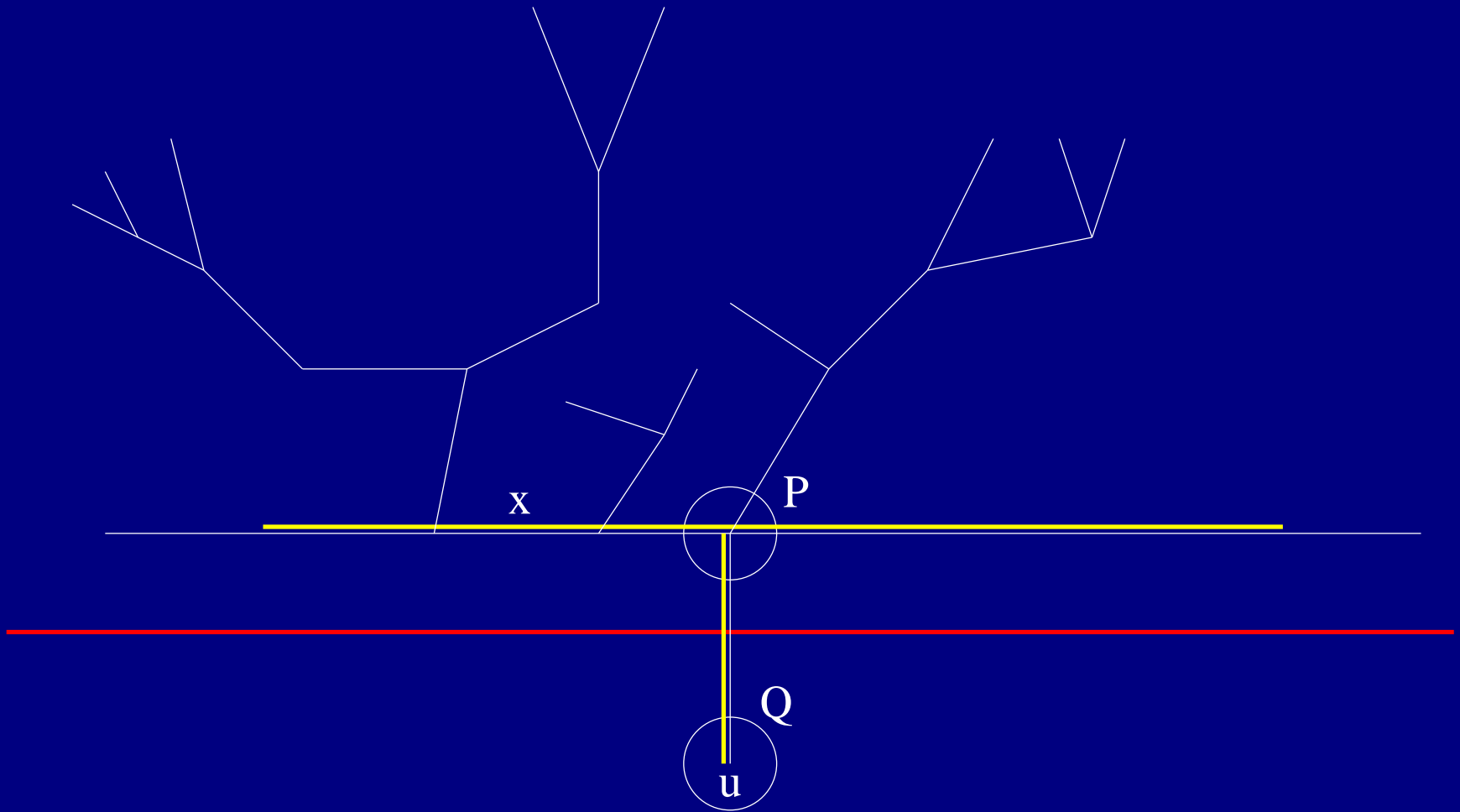
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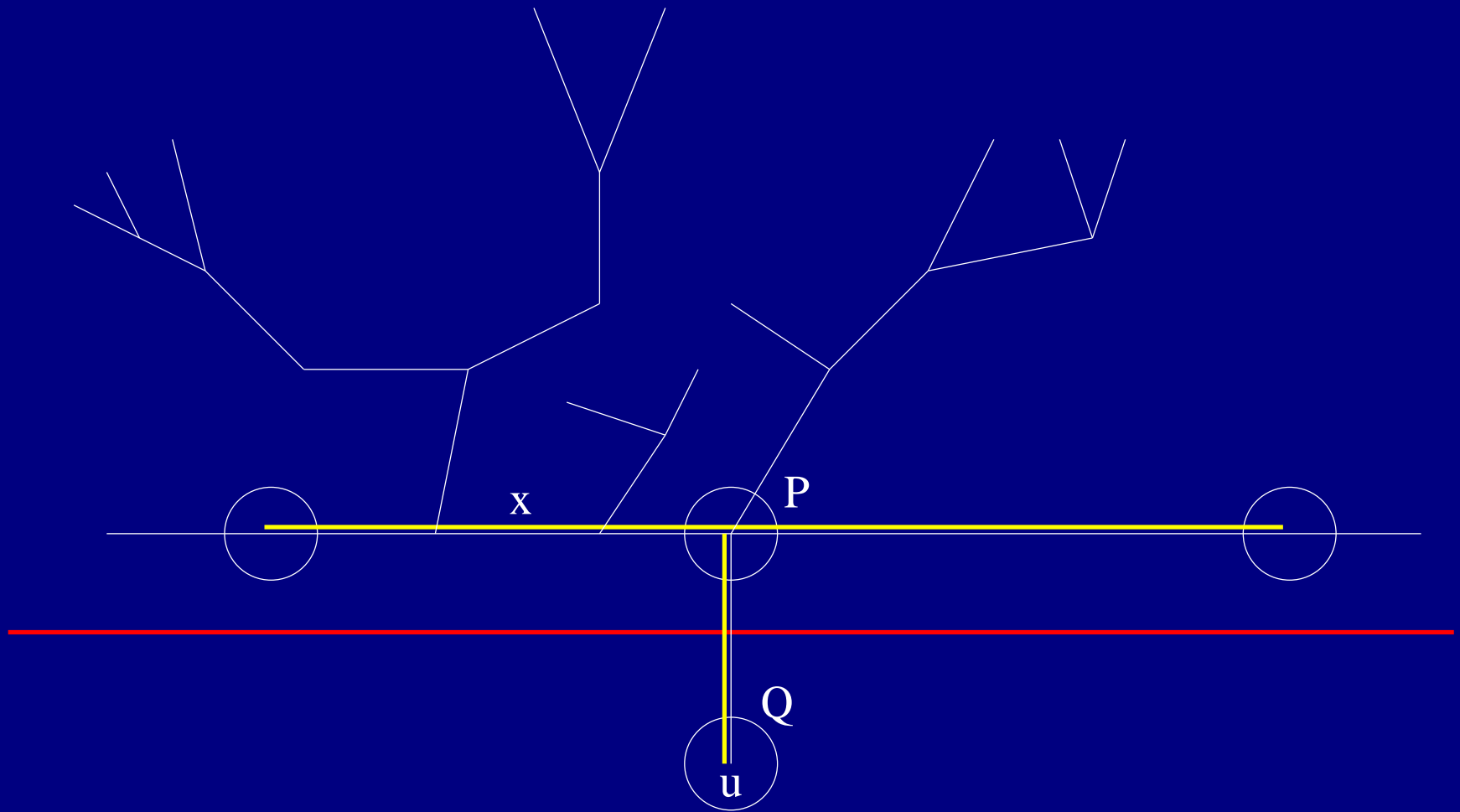
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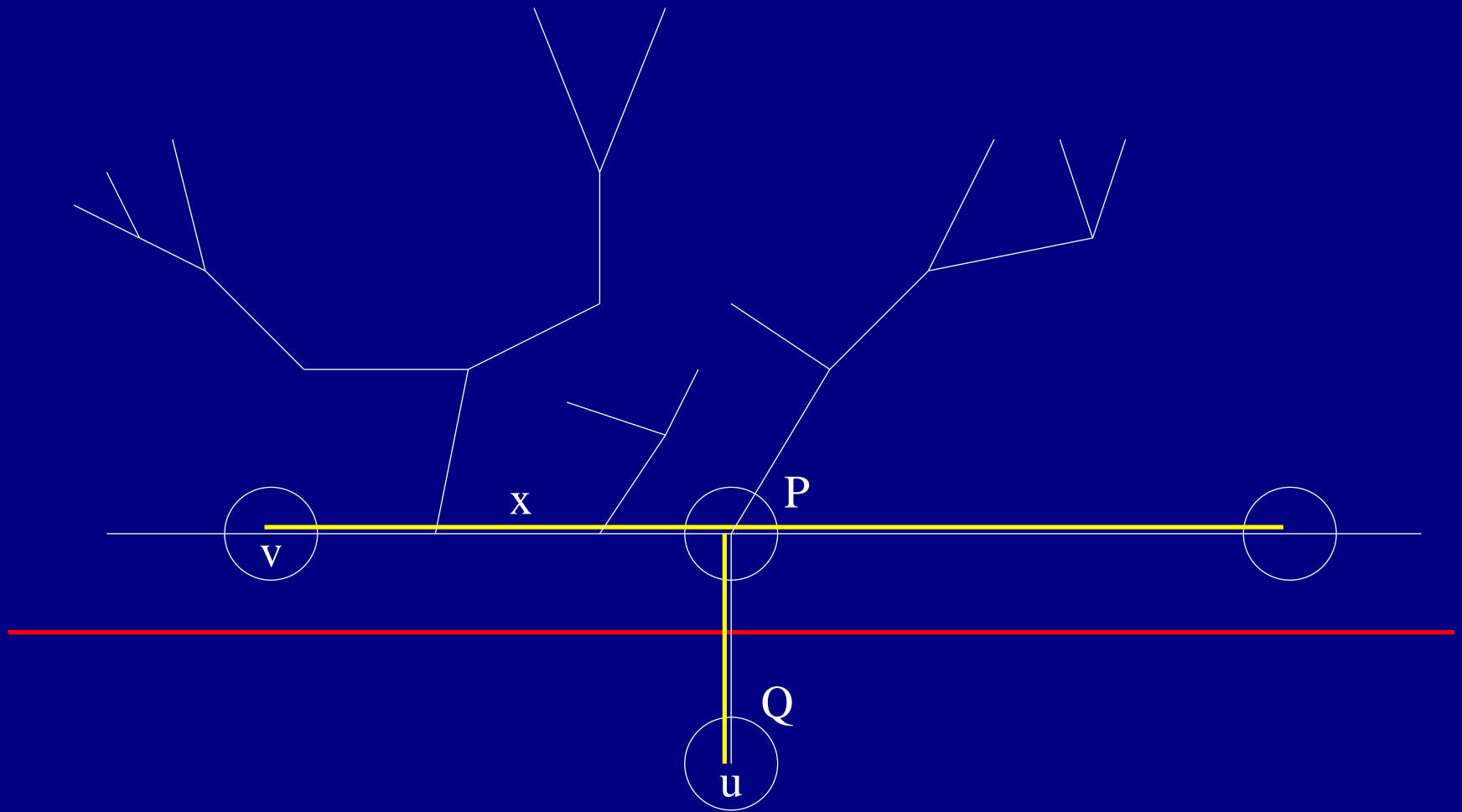
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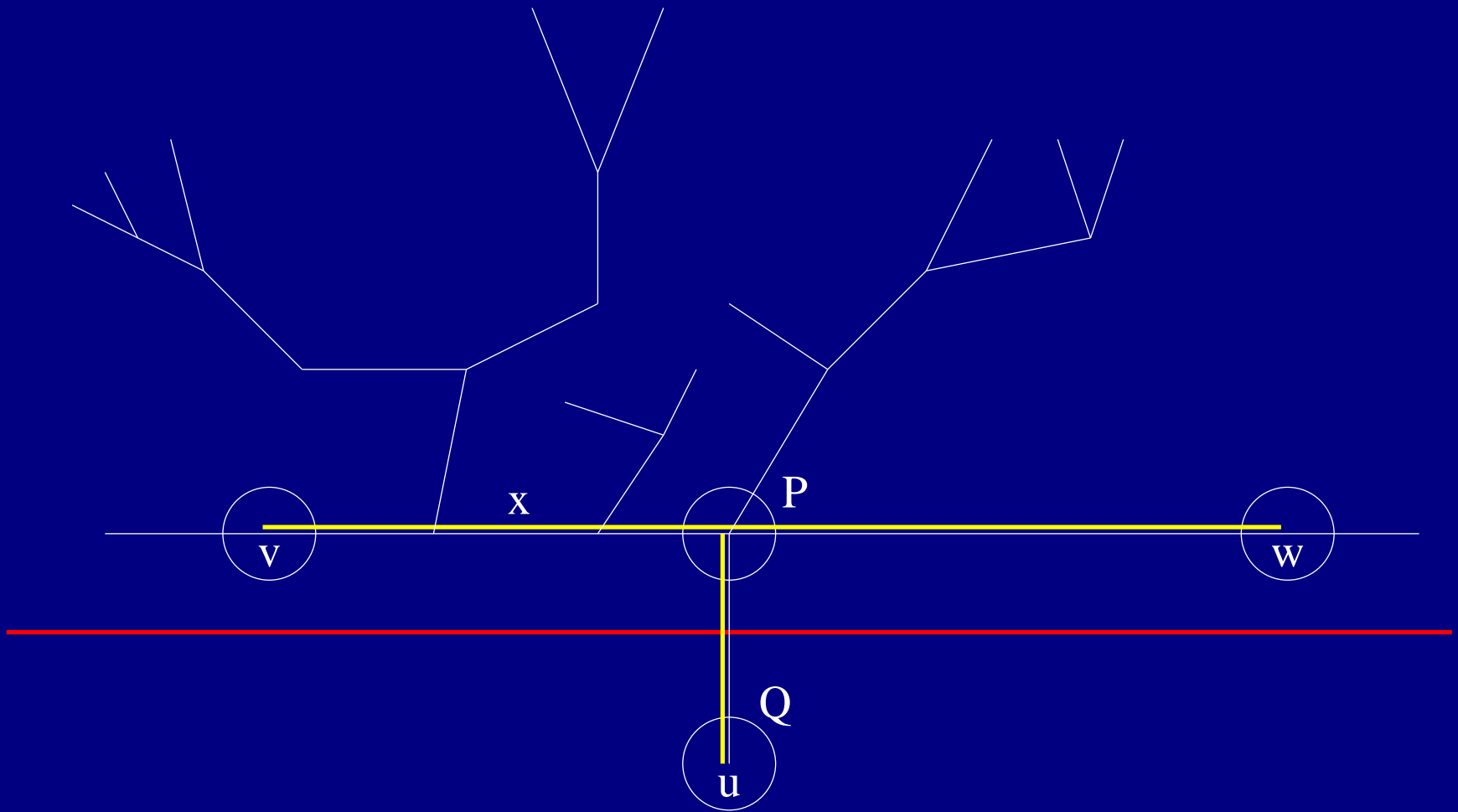
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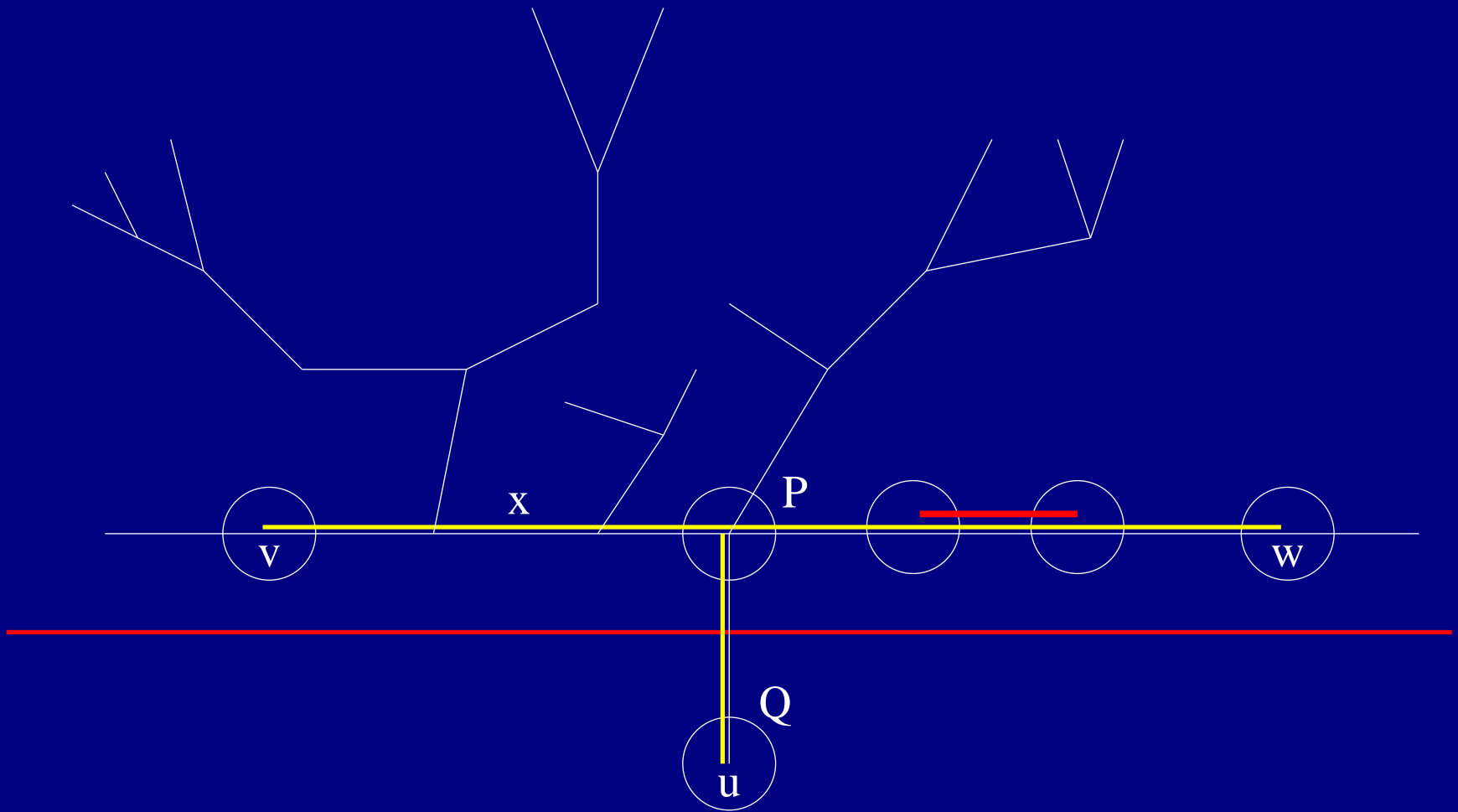
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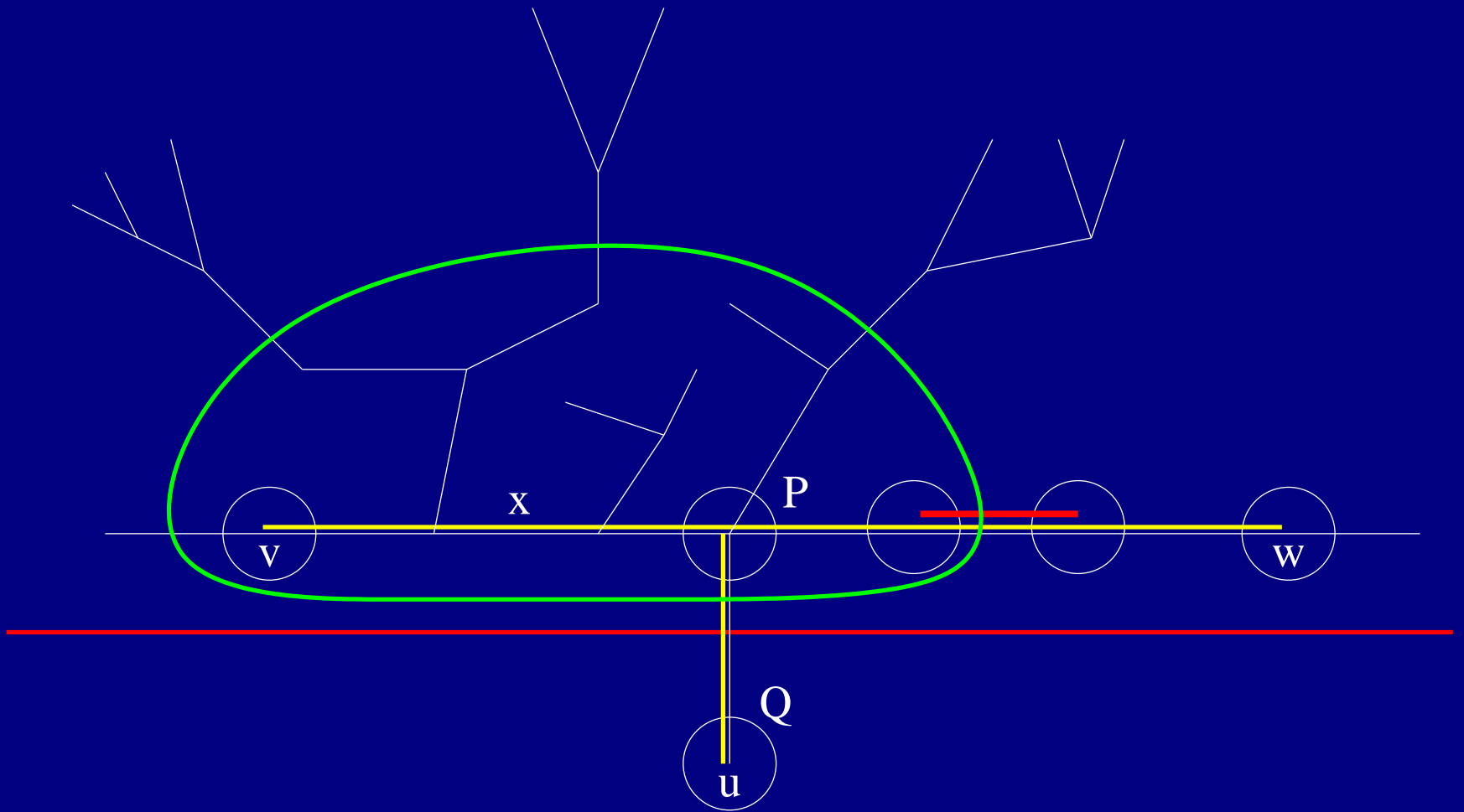
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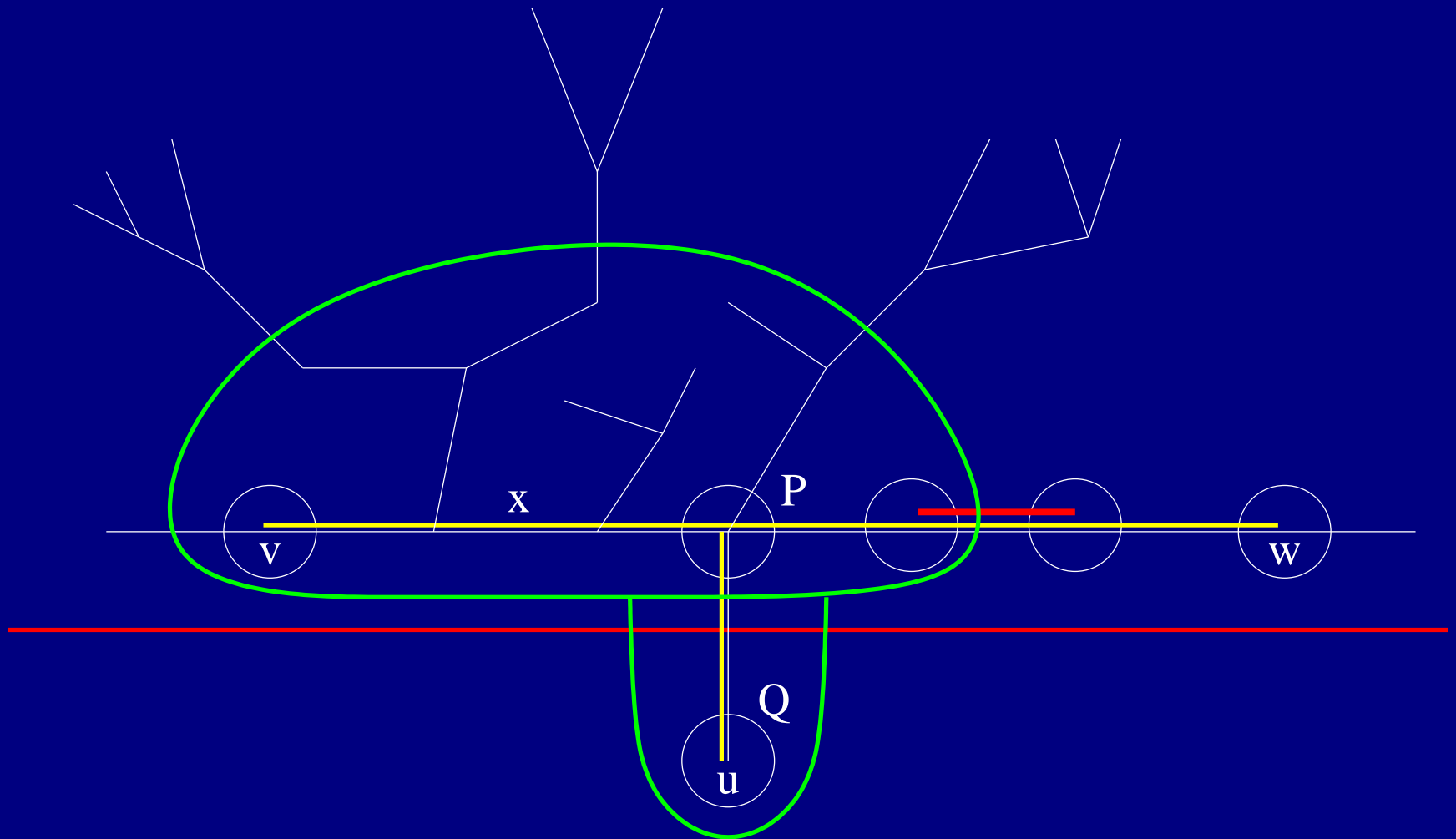
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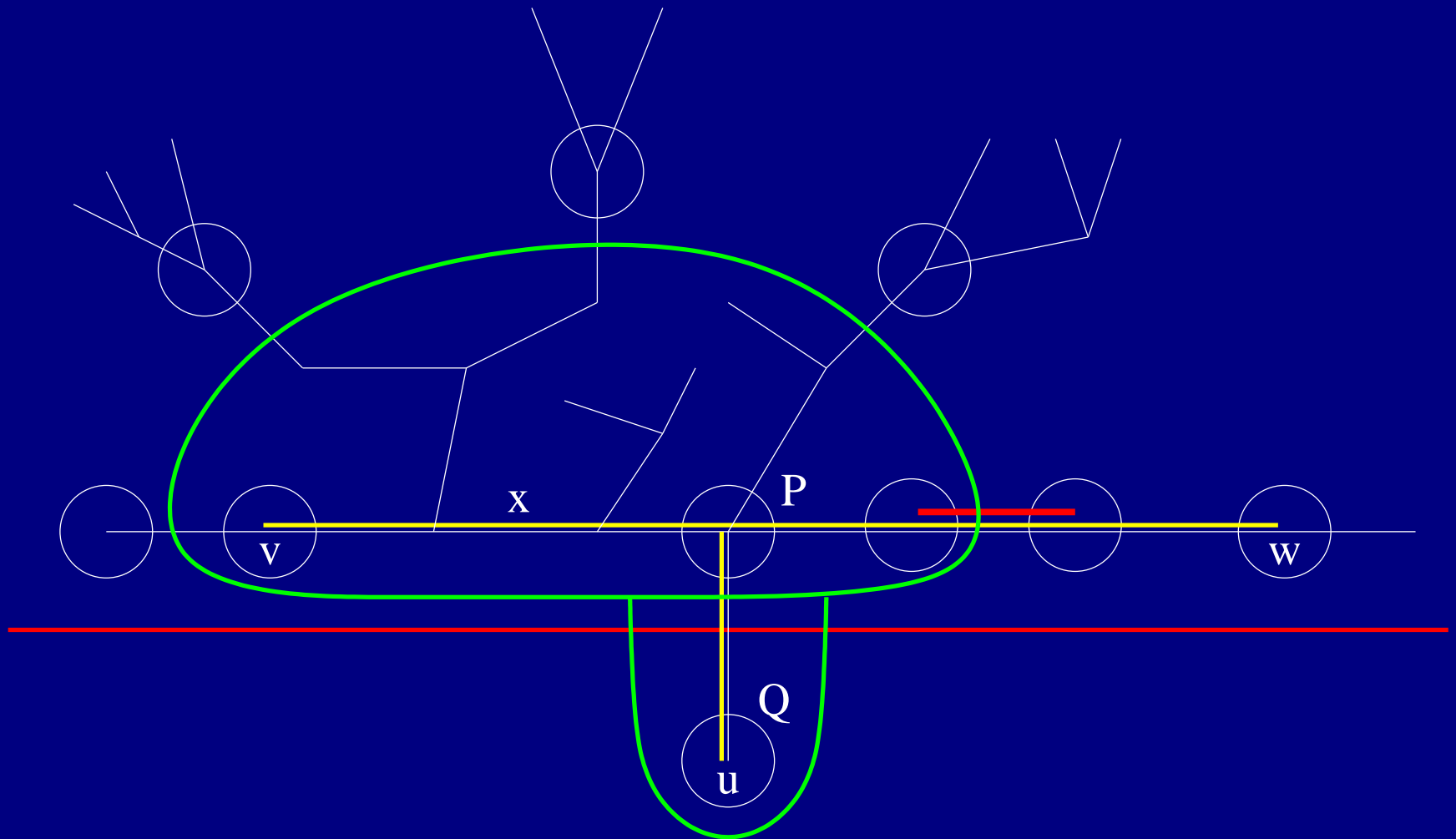
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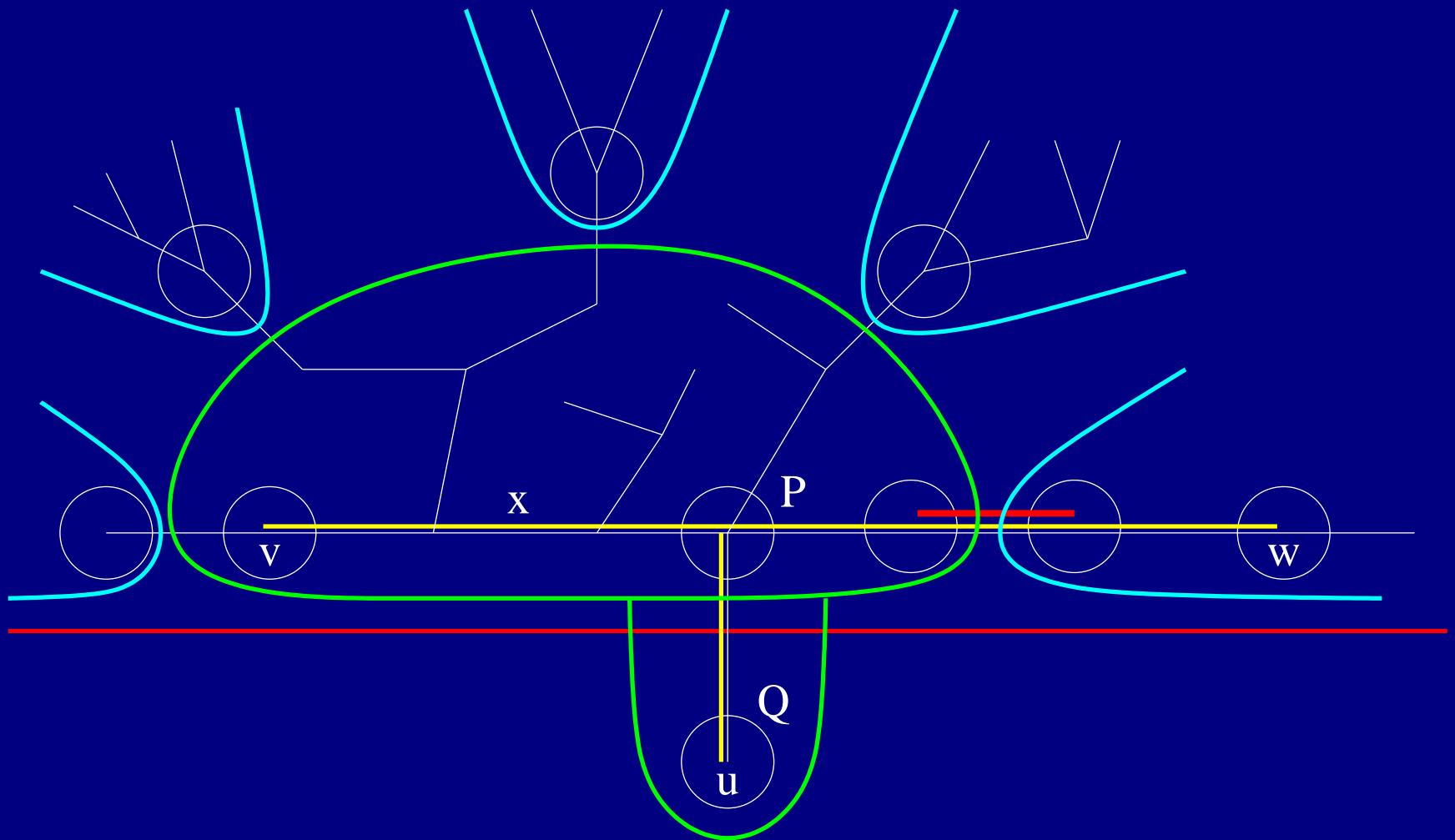
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Directed path graphs

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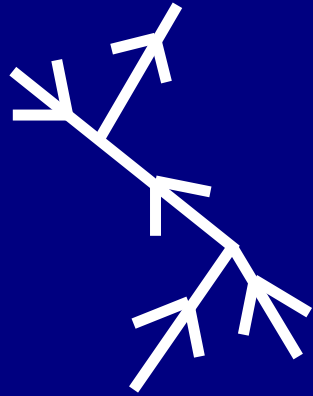
Definition

Intersection graph of a family of directed subpaths of a directed tree

Directed path graphs

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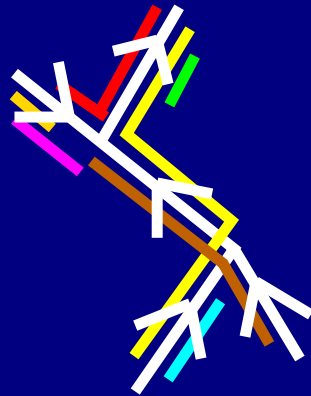
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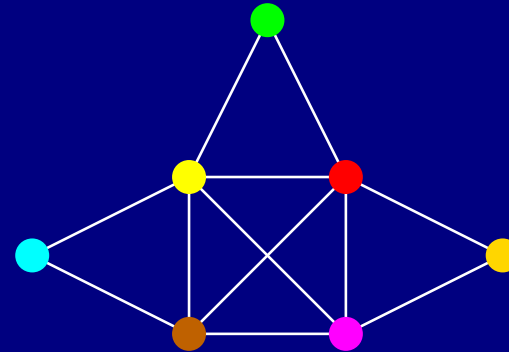
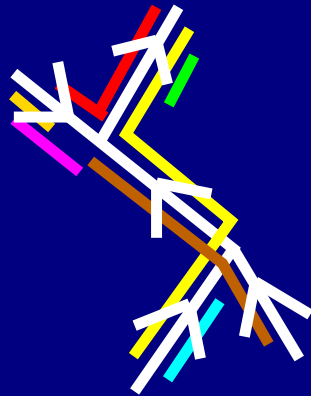
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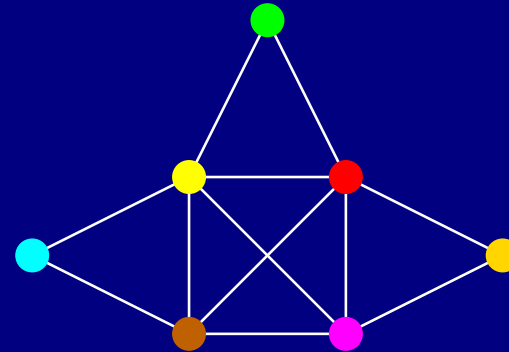
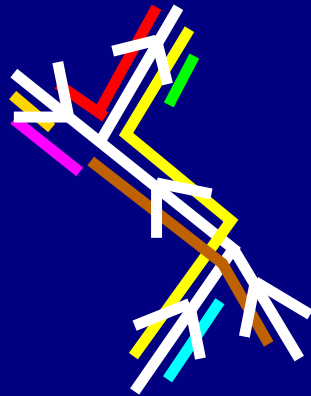
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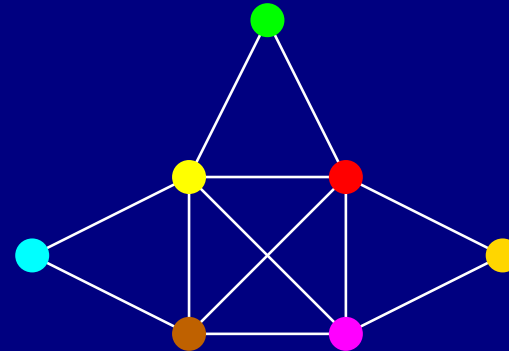
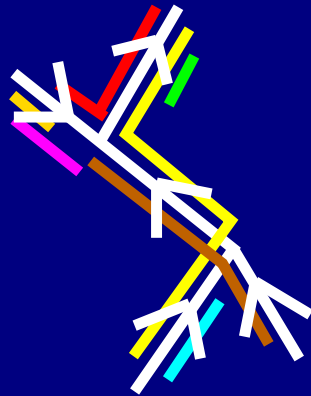
Theorem (Panda - 1999)

Directed path graph \iff it does not contain :

Directed path graphs

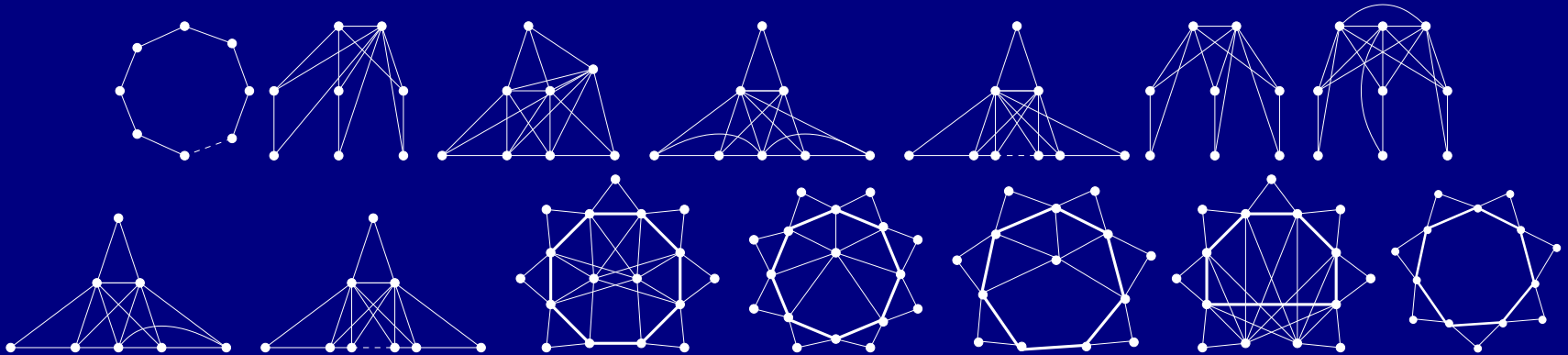
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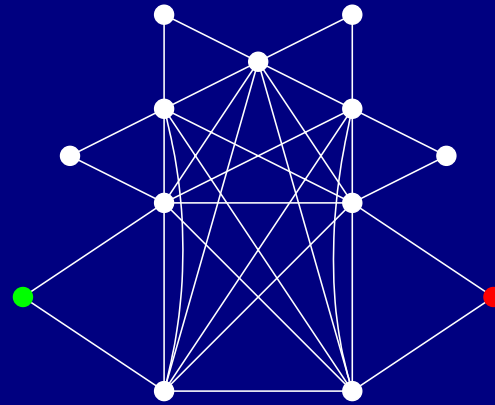
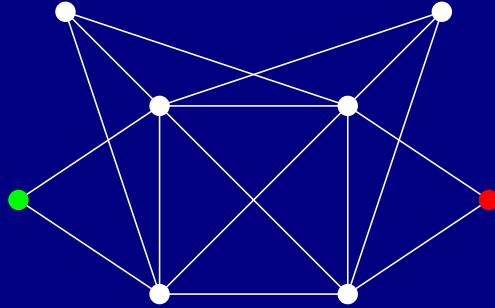
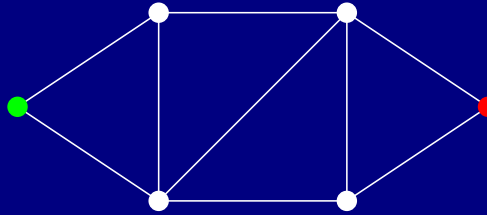
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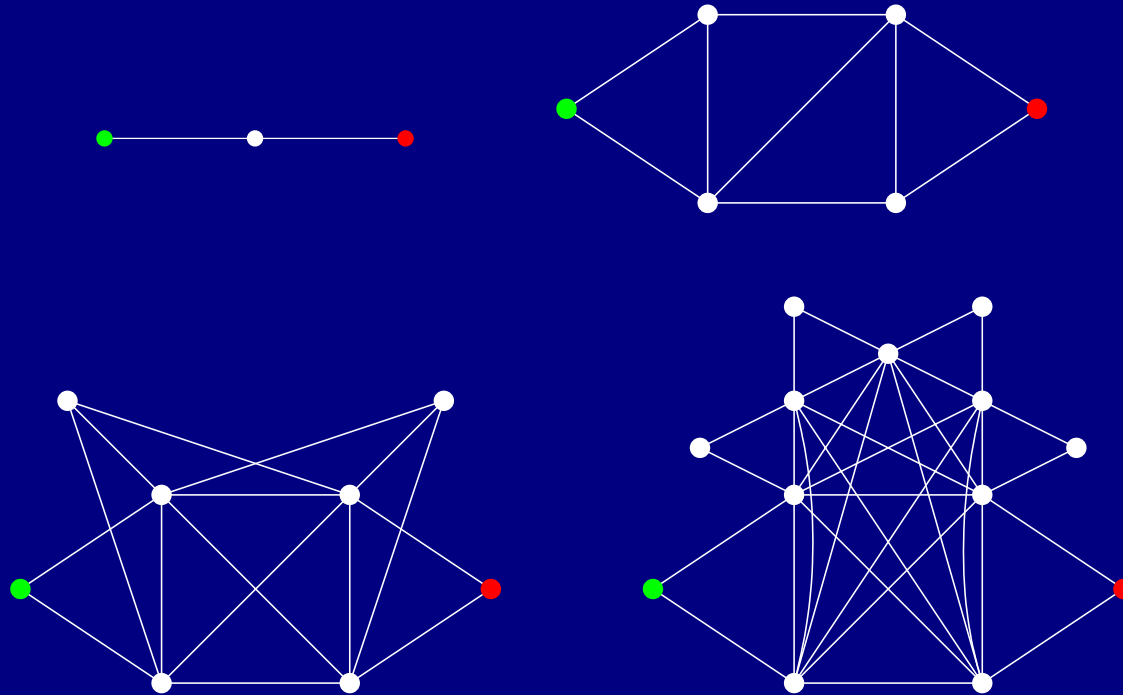


Strong path

Strong path



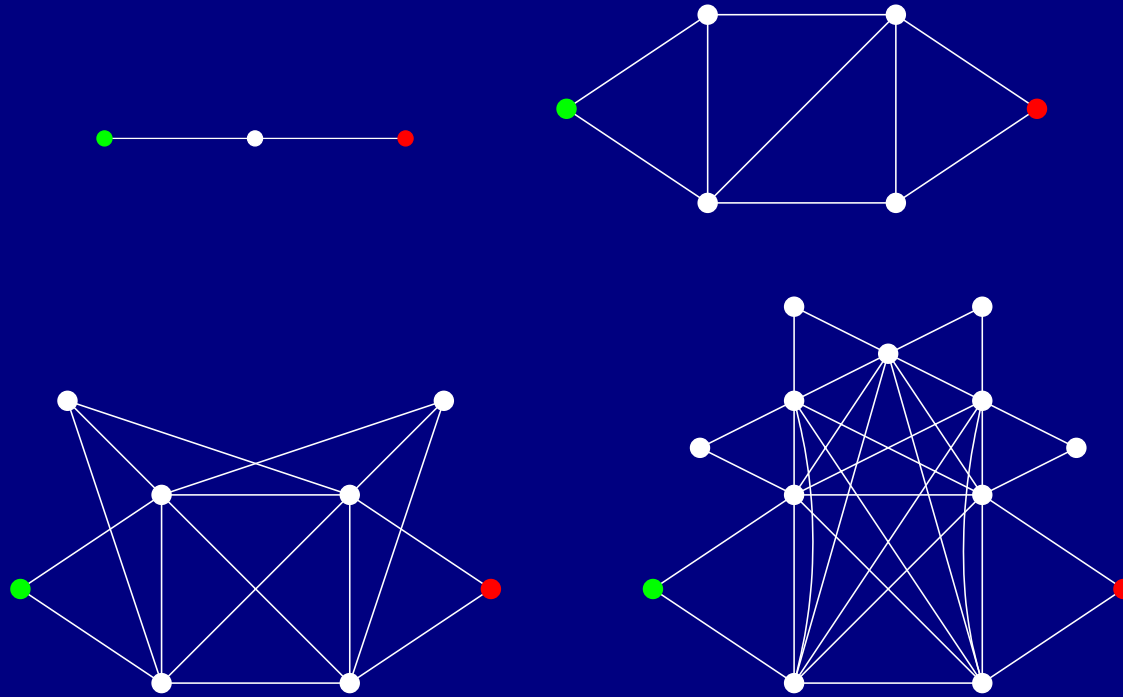
Strong path



Property

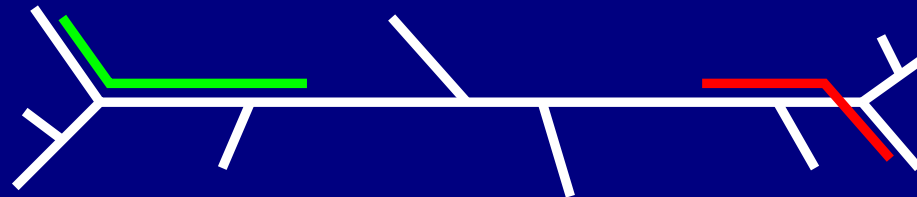
In any directed path graph model the path between two vertices that are linked by a strong path is directed.

Strong path

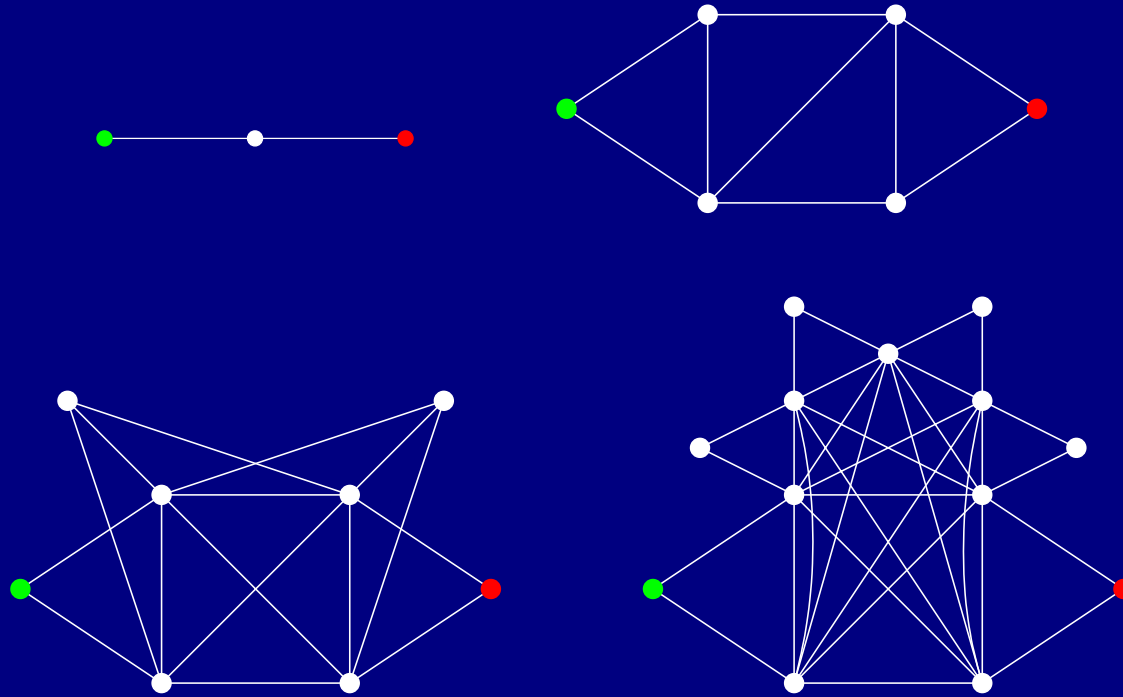


Property

In any directed path graph model the path between two vertices that are linked by a strong path is directed.

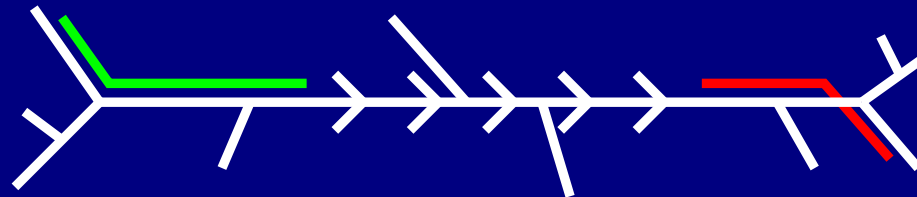


Strong path



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Strong asteroidal triple

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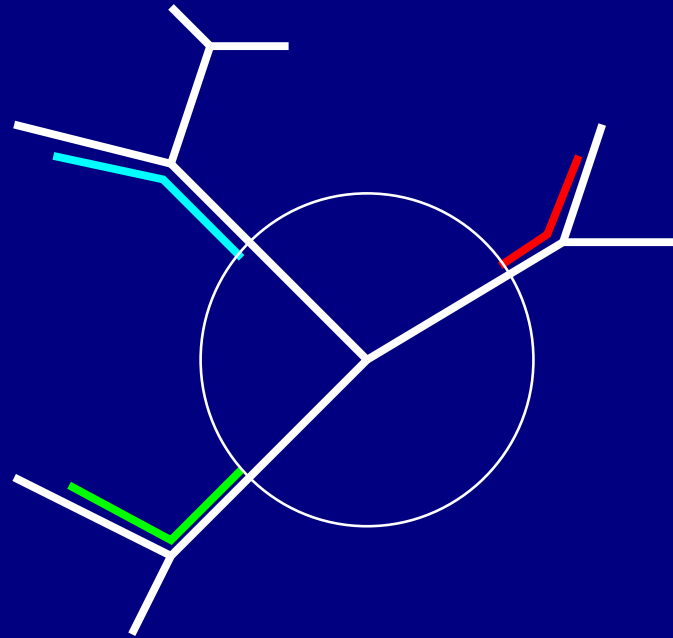
Definition

An asteroidal triple such that between any two of the vertices of the triple, there exists a strong path between them.

Strong asteroidal triple

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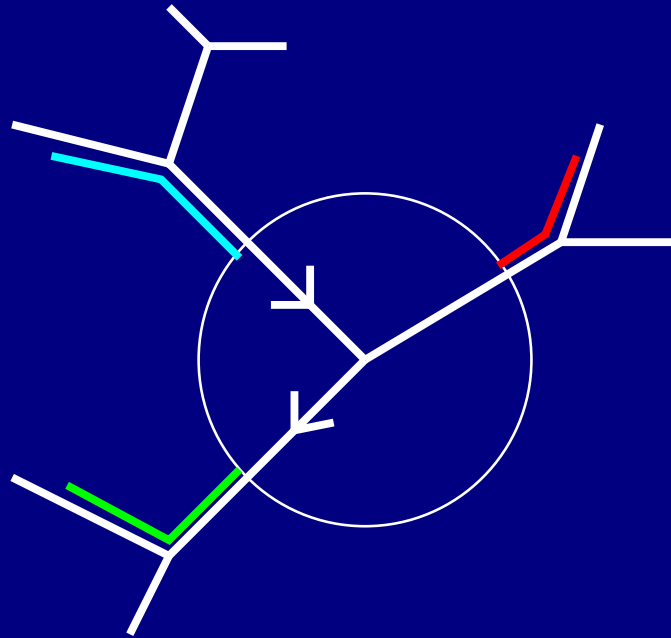
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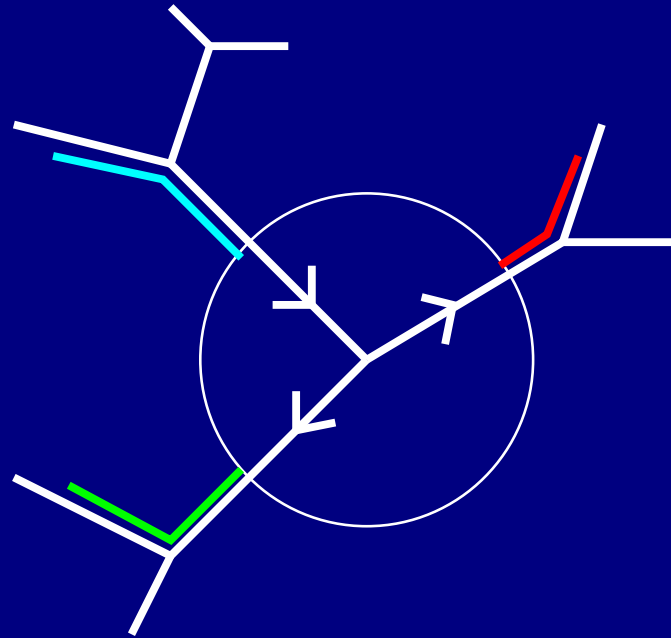
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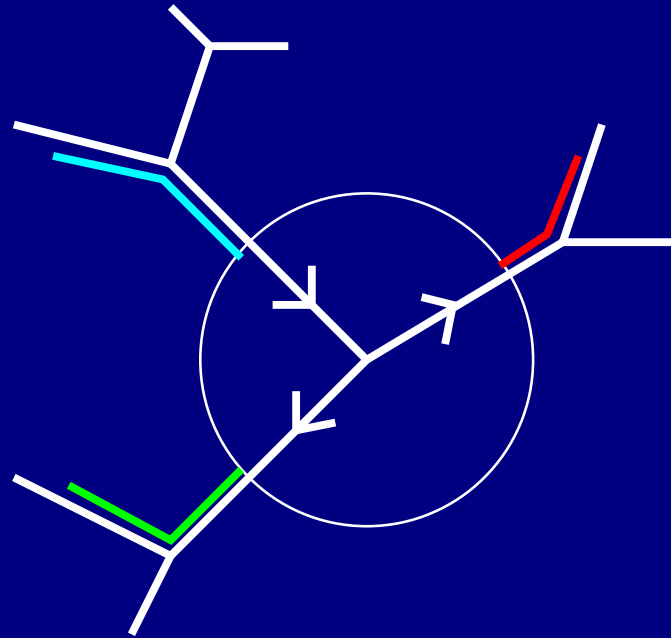


Theorem (Cameron, Hoàng, Lévêque - 2009)

Strong asteroidal triple

Definition

An asteroidal triple such that between any two of the vertices of the triple, there exists a strong path between them.



Theorem (Cameron, Hoàng, Lévêque - 2009)

Directed path graph \iff chordal + no strong asteroidal triple

Rooted path graphs

Definition

Intersection graph of a family of directed subpaths of a rooted tree

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Open problem

Forbidden induced subgraph characterization ?

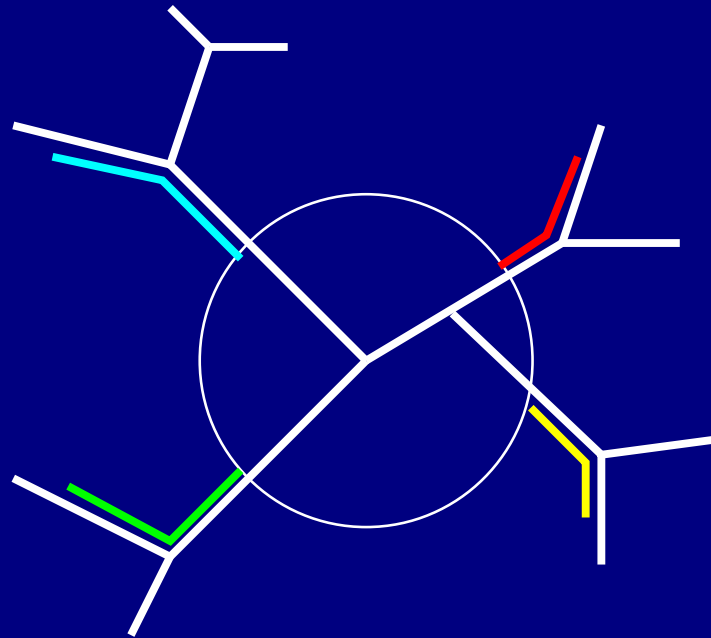
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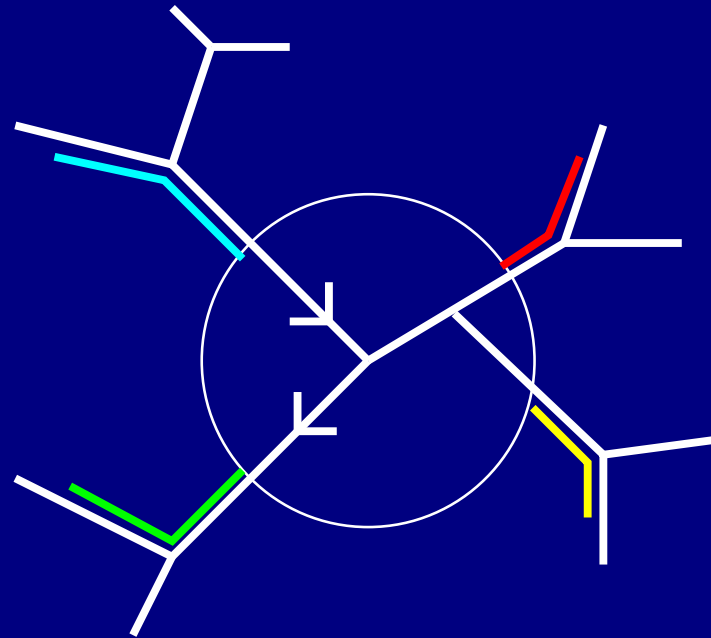
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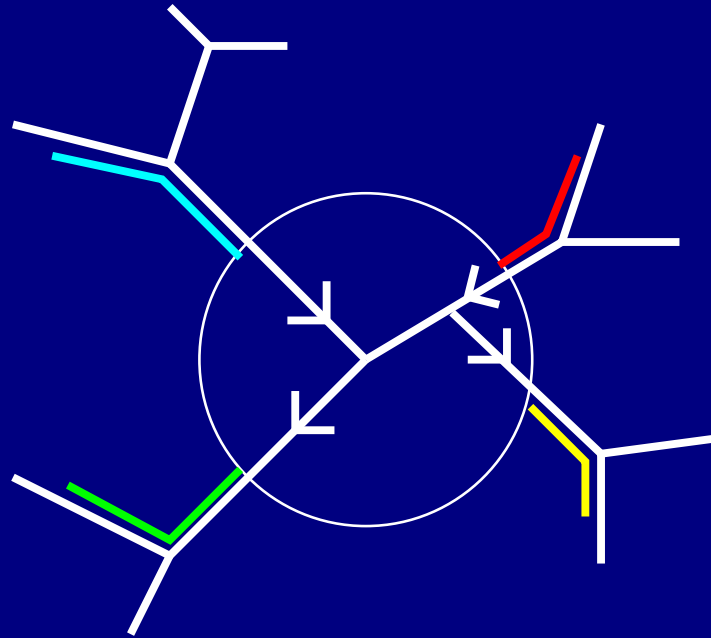
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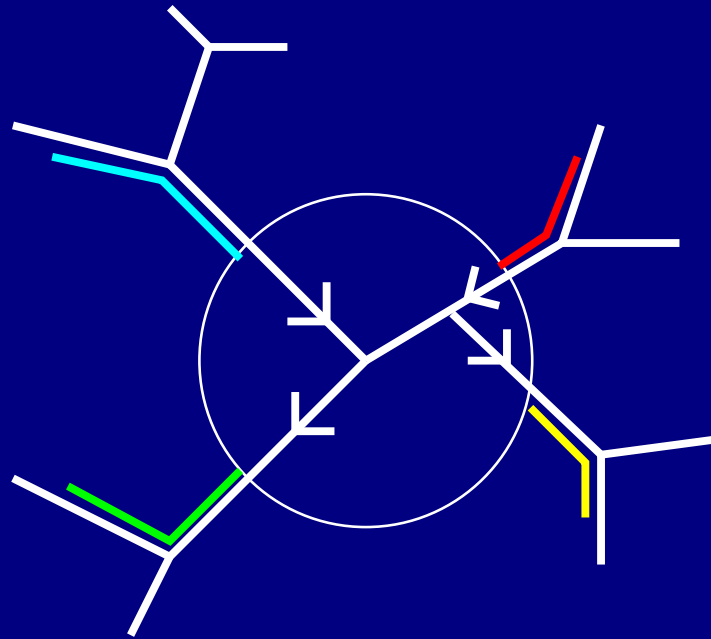
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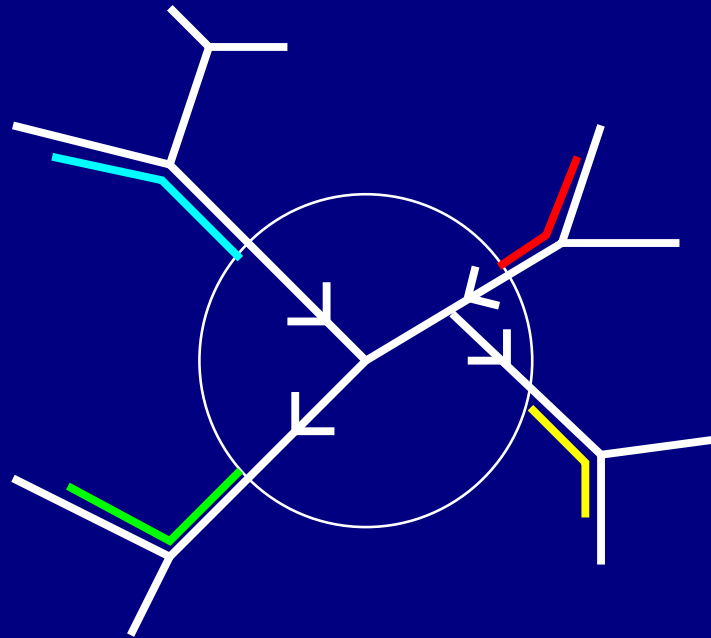
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Conjecture

Rooted path graph \iff

chordal + no strong asteroidal triple + no weak asteroidal quadruple

Summary of results and open problems

	forbidden subgraphs	forbidden asteroids
path graphs	[LMP09]	?
directed path graphs	[Pan99]	[CHL09]
rooted path graphs	?	?
interval graphs	[LB62]	[LB62]

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- Rooted path graphs : $\mathcal{O}(n + m)$ (Dietz - 1984) ??? thesis